Multilevel Growth Models
Individual Development Over Time

\[
\begin{align*}
\text{(1)} & \quad y_{ti} &= \eta_{0i} + \eta_{1i} x_t + \varepsilon_{ti} \\
\text{(2a)} & \quad \eta_{0i} &= \alpha_0 + \gamma_0 w_i + \zeta_{0i} \\
\text{(2b)} & \quad \eta_{1i} &= \alpha_1 + \gamma_1 w_i + \zeta_{1i}
\end{align*}
\]
Growth Modeling Approached In Two Ways: Data Arranged As Wide Versus Long

- **Wide: Multivariate, Single-Level Approach**

  \[ y_{ti} = i_i + s_i \times \text{time}_{ti} + \varepsilon_{ti} \]

  - \( i_i \) regressed on \( w_i \)
  - \( s_i \) regressed on \( w_i \)

- **Long: Univariate, 2-Level Approach (CLUSTER = id)**

  **Within**

  **Between**

  The intercept \( i \) is called \( y \) in Mplus
Growth Modeling Approached In Two Ways:  
Data Arranged As Wide Versus Long (Continued)

- Wide (one person):

  t1  t2  t3  t1  t2  t3

  Person i:  id  y1  y2  y3  x1  x2  x3  w

- Long (one cluster):

  t1  id  y1  x1  w
  t2  id  y2  x2  w
  t3  id  y3  x3  w
Three-Level Modeling In Multilevel Terms

Time point $t$, individual $i$, cluster $j$.

$y_{tij}$ : individual-level, outcome variable
$a_{1tij}$ : individual-level, time-related variable (age, grade)
$a_{2tij}$ : individual-level, time-varying covariate
$x_{ij}$ : individual-level, time-invariant covariate
$w_j$ : cluster-level covariate

Three-level analysis (Mplus considers Within and Between)

\begin{align*}
\text{Level 1 (Within)} : & \quad y_{tij} = \pi_{0ij} + \pi_{1ij} a_{1tij} + \pi_{2tij} a_{2tij} + e_{tij}, \\
& \quad \pi_{0ij} = \beta_{00j} + \beta_{01j} x_{ij} + r_{0ij}, \\
& \quad \pi_{1ij} = \beta_{10j} + \beta_{11j} x_{ij} + r_{1ij}, \\
& \quad \pi_{2tij} = \beta_{20tj} + \beta_{21tj} x_{ij} + r_{2tij}. \\
\text{Level 2 (Within)} : & \quad \beta_{00j} = \gamma_{000} + \gamma_{001} w_j + u_{00j}, \\
& \quad \beta_{10j} = \gamma_{100} + \gamma_{101} w_j + u_{10j}, \\
& \quad \beta_{20tj} = \gamma_{200t} + \gamma_{201t} w_j + u_{20tj}, \\
& \quad \beta_{01j} = \gamma_{010} + \gamma_{011} w_j + u_{01j}, \\
& \quad \beta_{11j} = \gamma_{110} + \gamma_{111} w_j + u_{11j}, \\
& \quad \beta_{21tj} = \gamma_{210} + \gamma_{21t} w_j + u_{2tj}.
\end{align*}
Two-Level Growth Modeling (Three-Level Modeling)

Within

Between
Further Readings On Three-Level Growth Modeling

