Introduction To Two-Level Regression Analysis With Continuous Outcomes
Two-level analysis (individual $i$ in cluster $j$):

$y_{ij}$ : individual-level outcome variable  
$x_{ij}$ : individual-level covariate  
$w_j$ : cluster-level covariate

Random intercepts, random slopes:

\[
\text{Level 1 (Within)} : y_{ij} = \beta_0 + \beta_1 x_{ij} + r_{ij}, \quad (1)
\]

\[
\text{Level 2 (Between)} : \beta_0 = \gamma_{00} + \gamma_{01} w_j + u_{0j}, \quad (2a)
\]

\[
\text{Level 2 (Between)} : \beta_1 = \gamma_{10} + \gamma_{11} w_j + u_{1j}. \quad (2b)
\]

• Mplus gives the same estimates as HLM/MLwiN ML (not REML):  
  • $V(r)$ (residual variance for level 1)  
  • $\gamma_{00}, \gamma_{01}, \gamma_{10}, \gamma_{11}, V(u_0), V(u_1), Cov(u_0, u_1)$  
• Centering of $x$: subtracting grand mean or group (cluster) mean
NELS Data

• The data—National Education Longitudinal Study (NELS:88)

  • Base year Grade 8—followed up in Grades 10 and 12

  • Students sampled within 1,035 schools—approximately 26 students per school, n = 14,217

  • Variables—reading, math, science, history-citizenship-geography, and background variables
NELS Math Achievement Regression

**Within**

- female
- stud_ses
- s1
- s2
- m92

**Between**

- per_adva
- private
- catholic
- mean_ses
- s1
- s2
- m92
TITLE: NELS math achievement regression

DATA: FILE IS completev2.dat;
   ! National Education Longitudinal Study (NELS)
   FORMAT IS f8.0 12f5.2 f6.3 f11.4 23f8.2
   f18.2 f8.0 4f8.2;

VARIABLE: NAMES ARE school r88 m88 s88 h88 r90 m90 s90 h90 r92
       m92 s92 h92 stud_ses f2pnlwt transfer minor coll_asp
       algebra retain aca_back female per_min hhw_time
       salary dis_fair clas_dis mean_col per_high unsafe
       num frie teaqual par_invo ac_track urban size rural
       private mean_sess catholic stu_teac per_adva tea_exce
       tea_res;

USEV = m92 female stud_ses per_adva private catholic
       mean_sess;

   !per_adva = percent teachers with an MA or higher

WITHIN = female stud_ses;
BETWEEN = per_adva private catholic mean_sess;
MISSING = blank;
CLUSTER = school;
CENTERING = GRANDMEAN (stud_sess per_adva mean_sess);
ANALYSIS: TYPE = TWOLEVEL RANDOM MISSING;

MODEL:

%WITHIN%
s1 | m92 ON female;
s2 | m92 ON stud_ses;

%BETWEEN%
m92 s1 s2 ON per_adva private catholic mean_ses;
m92 WITH s1 s2;

OUTPUT: TECH8 SAMPSTAT;
Output Excerpts NELS Math Achievement Regression

N = 10,933

Summary of Data

Number of clusters 902

Size (s) Cluster ID with Size s

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Output Excerpts NELS Math Achievement Regression (Continued)

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Average cluster size  12.187

Estimated Intraclass Correlations for the Y Variables

Intraclass Variable Correlation

M92    0.107
Output Excerpts NELS Math Achievement Regression (Continued)

Tests of Model Fit

Loglikelihood

H0 Value -39390.404

Information Criteria

Number of Free parameters 21
Akaike (AIC) 78822.808
Bayesian (BIC) 78976.213
Sample-Size Adjusted BIC 78909.478
(n* = (n + 2) / 24)

Model Results

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<tr>
<th>Estimates</th>
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<th>Est./S.E.</th>
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### Output Excerpts NELS Math

**Achievement Regression (Continued)**

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**Intercepts**

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**Residual Variances**

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Cross-Level Influence

Between-level (level 2) variable \( w \) influencing within-level (level 1) \( y \) variable:

**Random intercept**

\[
y_{ij} = \beta_{0j} + \beta_1 x_{ij} + r_{ij}
\]

\[
\beta_{0j} = \gamma_{00} + \gamma_{01} w_j + u_{0j}
\]

Mplus:

```plaintext
MODEL:
%WITHIN%;
y ON x;   ! estimates beta1
%BETWEEN%;
y ON w;   ! y is the same as beta0
            ! estimates gamma01
```
Cross-level interaction, or between-level (level 2) variable moderating a within level (level 1) relationship:

Random slope

\[ y_{ij} = \beta_0 + \beta_{1j} x_{ij} + r_{ij} \]
\[ \beta_{1j} = \gamma_{10} + \gamma_{11} w_j + u_{1j} \]

Mplus:

```
MODEL:
  %WITHIN%;
  betal | y ON x;
  %BETWEEN%;
  betal ON w;      ! estimates gamma11
```
Generalizations Of Two-Level Regression

The Mplus framework allows random slopes for

- Observed covariates
- Observed dependent variables (for example, mediational modeling with random slopes)
- Factors
Further Readings On
Multilevel Regression Analysis


See also
